JORDON FORM CONTINUED...

$$Y(s) = b_0 U(s) + \frac{c_1 U(s)}{(s+P_1)^3} + \frac{c_2 U(s)}{(s+P_1)^2} + \ldots$$

$$X_1(s) = \frac{U(s)}{(s+P_1)^3}$$

$$X_2(s) = \frac{U(s)}{(s+P_1)^2}$$

$$X_3(s) = \frac{U(s)}{(s+P_1)}$$

$$X_4(s) = \frac{U(s)}{s+P_4}$$

$$\frac{X_1(s)}{X_2(s)} = \frac{1}{s+P_1} \Rightarrow X_2(s) = sX_1(s) + P_1 X_1(s)$$

likewise

$$sX_1(s) = X_2(s) - P_1 X_1(s)$$

$$\mathcal{L}^{-1} \quad \dot{x}_1 = x_2 - P_1 x_1$$

$$\dot{x}_2 = x_3 - P_1 x_2$$

$$\frac{X_3(s)}{U(s)} = \frac{1}{s+P_1} \Rightarrow sX_3(s) = -P_1 X_3(s) + U(s)$$

$$\mathcal{L}^{-1} \quad \dot{x}_3 = -P_1 x_3 + u$$

$$\dot{x}_4 = -P_4 x_4 + u$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Jordon block

$$A = \left[\begin{array}{ccc|ccc} -p_1 & 0 & 0 & & & \\ 0 & -p_1 & 0 & & \text{zeros} & \\ 0 & 0 & -p_1 & & & \\ \hline & & & -p_4 & 0 & 0 \\ & \text{zeros} & & 0 & \ddots & 0 \\ & & & 0 & 0 & -p_n \end{array}\right]$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$y = c_1 x_1 + c_2 x_2 + \cdots$$

$$C = [c_1 \; \cdots \; c_n] \qquad D = b_0$$

<u>EX</u>:

$$\frac{Y(s)}{U(s)} = \frac{s+3}{s^2+3s+2}$$

<u>controllable</u> canonical form.

$$D = 0 \qquad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = [3 \; 1]$$

observable canonical form

$$D=0 \quad A=\begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \quad B=\begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad C=\begin{bmatrix} 0 & 1 \end{bmatrix}$$

jordon canonical form

$$\frac{Y(s)}{u(s)} = \frac{c_1}{s+1} + \frac{c_2}{s+2}$$

$$c_1 = 2 \qquad c_2 = -1$$

$$D=0 \quad A=\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \quad C=\begin{bmatrix} 2 & -1 \end{bmatrix} \quad B=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

DIAGONALIZATION OF AN $n \times n$ MATRIX

$A \in \mathbb{R}^{n \times n}$

$A_1 \ldots A_n$ are the eiger values of A

$P_1 \ldots P_n$ are the eiger vectors associated with $d_1 \ldots d_n$.

P_i is the eiger vector associated with d_i

$P = [P_1 \ldots P_n] \in \mathbb{R}^{n \times n}$

$AP_i = d_i P_i$

$$P^{-1}AP = \begin{bmatrix} d_1 & & \emptyset \\ & \ddots & \\ \emptyset & & d_n \end{bmatrix}$$

let A be cont form.

$$A = \begin{bmatrix} 0 & 1 & & & \\ & & \ddots & & 0 \\ & 0 & & \ddots & \\ & & & & 1 \\ -a_n & \cdots & \cdots & \cdots & -a_1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & \cdots & \cdots & 1 \\ d_1 & d_2 & \cdots & d_n \\ d_1^2 & d_2^2 & \cdots & d_n^2 \\ \vdots & & & \\ d_1^{n-1} & & & d_n^{n-1} \end{bmatrix}$$

$d_1 \ldots d_n$ are real distinct.

$$z = P^{-1}x$$

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}}$$

$$\dot{z} = P^{-1}\dot{x} = P^{-1}[Ax + Bu]$$

$$= P^{-1}APz + P^{-1}Bu$$

$$y = CPz + Du$$

$$\boxed{\begin{aligned} \dot{z} &= (P^{-1}AP)z + P^{-1}Bu \\ y &= CPz + Du \end{aligned}}$$

$$P^{-1}AP = \begin{bmatrix} d_1 & & 0 \\ & \ddots & \\ 0 & & d_n \end{bmatrix}$$

FROM SS TO TF

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

assume $x(0) = 0$

$$s(x(s)) = A\,x(s) + B\,u(s)$$
$$(sI - A)\,x(s) = B\,u(s)$$
$$x(s) = (sI - A)^{-1} B\,u(s)$$

$$y(s) = C(sI - A)^{-1} B\,u(s) + D\,u(s)$$

$$\boxed{\frac{y(s)}{u(s)} = C(sI - A)^{-1}B + D}$$

Transfer Function

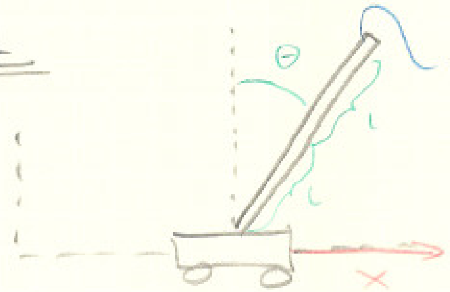
note: $$(sI - A)^{-1} = \frac{\mathrm{Adj}(sI - A)}{\det(sI - A)}$$

$$\boxed{\frac{y(s)}{u(s)} = \frac{C\,\mathrm{Adj}(sI - A)B + D\det(sI - A)}{\boxed{\det(sI - A)}}}$$

Transfer function again.

poles of transfer function.

HW# 1



CH 11

B11 1→5

* obtain the SS of this system